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Coretractable modules weakly

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Abstract: If R is an identity ring, and M is an R-Module on the right. This is the weakly coretractable module class. Any basic properties are analyzed and some connexions are formed between these and other similar modules.

Introduction

R is a unitary ring and all modules are unitary R-module in this article. There emerged the concept of a coretractable module. However, Amini studied this definition as a dual retrievable model "It's said that 'M' is coretractable if there's a non-null mapping of HomR(M/N,M)' in which 'R-module M' is considered a strongly coretractable module if there's a non-nuclear R-homomorphism f:M/NM for each sub module of N of M"Equally, "M shall be firmly interspersed if there is a null mapping f as End R(M) for any of the proper sub modules N of M". "The Z-module Z4, for example, is retractable but not coretractable and so generalizations are introduced." We will be able to use the Z-module Z3 as a coretractive module". In the notion of a lowly coretractable module, we add the notion that R-module M is regarded weakly as coretractable, when a non-null mapping falsifyEndR(M) is present as f2(K)=0. Clearly, a coretractable can be weakly rectified, but not the other way round. In Section One, some of the essential characteristics of coretractable modules are echoed. We have also applied some recent observations (to our understanding). In section two the weakly coretractable modules were discussed and analyzed. There have also been several similarities between it and other modular groups.

On Coretractable Modules

Note the meaning as follows:

Definition: R-module M is referred to as coretractable if there is a non-zero R-homorphism f: M / S alternately M for each proper S sub module S.'

1) A module M is coretractable if ∀S<M, ∃ 0≠f∈EndR(M) such that f(S)=0; S⊆kerf.

Proof: (⟹) Suppose that S<M, ∃ f:M→M, f≠0 and f(S)=0. Define g:M/S→M by g(m+S)=f(m), it isclear that g is well-defined R-homomorphism. Now, since f≠0 and f(S)=0, then ∃m∈M, m∉S suchthat f(m) ≠0. Thus g(m+S)=f(m) ≠0. Thus HomR(M/S, M)≠0 ∀S<M.(⟸)Let S<M, since ∃ f:M/S→M, f≠0. It follows that f∘π∈EndR(M) and f∘π(S)= f(π(S))=f(0)=0,where π is the natural epimorphism from M into M/S. Thus M is coretrectable.

2) "Clearly every semisimple module is coretractable, and hence every R-module over asemi simple ring is coretractable.But it may be that coretractable module not semisimple as the Z-module Z4.
3) "Zp∞ is coretractable Z-module, since Zp∞/S≅Zp∞ (∀S<Zp∞)".
4) "Zn is coretractable Z-module (∀n∈Z+)" [2].
5) See the Z-module Q. Suppose Q is a coretractable module. Since Z<Q, ∃f ∈EndR(Q), f≠0with f(Z)=0. Now, for any s/t∈Q, f(s/t)= f(1/t) s. But 0=f(1)= f(1/t) s, so f(1/t)=0 and hencef(s/t)=0. t=0; f=0, which is a contradiction. Thus Q is not coretractable module.
6) See M=Z⨁Z2 as Z-module. Let N=3Z ⨁Z2, M/N≅Z3. But there is no nonzero mapping f:Z3→ Z⨁Z2, since if we assume there exists $\bar{x}$ ∈Z3 and f($\bar{x}$)≠(0, $\bar{0}$). So if f($\bar{x}$)=(n, $\bar{m}$), n≠0, then f($\bar{x}$.3)=f($\bar{0}$)=(0, $\bar{0}$). On the other hand , f($\bar{x}$. 3)= f($\bar{x}$). 3= (n, $\bar{m}$). 3= (n. 3, $\bar{m}$. 3) ≠(0, $\bar{0}$) which is a contradiction . Similarly if f ($\bar{x}$)=(0, $\bar{1}$), then f($\bar{x}$. 3)=f($\bar{0}$)=(0, $\bar{0}$), but f($\bar{x}$). 3=(0, $\bar{1}$). 3≠(0, $\bar{0}$) which is a contradiction. Thus Z⨁Z2 is not coretrectable.
7) See M=Z⨁Z as Z-module. Let N=Z⨁2Z suppose ∃ g∈EndZ(M), g≠0, g(N)=0 then g(a, b)=0 ∀a∈Z, b∈2Z. Now, for all odd integers x and y, x=2m+1 and y= 2n +1 for some n, m∈Z, g(x, y)= g(2m+1, 2n+1)= g(2m+1,2n)+g(0,1)= 0+ g(0,1)= g(0,1), but g(0, 2)= g(0, 1)2, then g(0, 1)=0, hence g(x,y)=0. Now for all even integer 2m and odd integer 2n+1; g(2m,2n+1)=g(2m,2n)+g(0,1)=0. Thus g(M)=0 that is g is a zero mapping which is acontradiction. Thus M is not coretractable module.
8) Coretractability is preserved by an isomorphism.

Recall that "a submodule B of M is relative complement of A in M if B is maximal with respect to the property B∩A=0". "A submodule E of M is essential if ∀ W≤M if E∩W=0, then W=0(denoted E≤eM)". The following Proposition appears in without proof.

Proposition 3: An R-module M is coretractable if HomR(M/E, M)≠0 ∀E≤eM.

Proof: It is clear.

Proposition 4: An R -module M is coretractable if M is coretractable $\bar{R}$ -module (where $\bar{R}$ =R/annM).

Proof: Since every R -submodule of M is $\bar{R}$ -submodule of M and conversely , also every R -homomorphismis an $\bar{R}$-homomorphism and conversely. Hence the result follows directly.

"A module M is cogenerator if every nonzero homomorphism f:M1→ M2 where M1 and M2 aremodules, there is g:M2→M such that g∘f ≠ 0 [5, P. 507] and [6, P. 53]. Equivalently an R-module Mis called a cogenerator if for any R-module N and 0≠x∈N, there exists g: N→M such that g(x)≠0" [5,P. 507].

Proposition 5: Every cogenerator R-module is coretractable module.

Proof: Let M be a cogenerator module and let S<M, then M/S≠(0). Let x=m+S≠0, ∃g:M/S →M suchthat g(x) ≠0, that is g≠0 and hence M is coretractable module.

The converse of Proposition5 may be not true. Consider Z2 is coretractable(by part (4)), but itis not cogenerator module. Since the only nonzero mapping from Z into Z2 is given by:

g(x) ={$\bar{0}$if x is even integer,$\bar{1}$if x is odd integer

Thus for each nonzero even integer x, there is no g: Z→Z2 such that g(x)≠0.

Corollary: For an R-module M, if Πi∈ΛM is a cogenerator module, then M is coretractablemodule.

Proof: By [5, Corollary19.7, P. 508], M is cogenerator and hence by Proposition, M iscoretrectable.

Corollary: If R is cogenerator. Then any faithful R-module is coretrectable.

Proof: Take M is faithful R-module. Then by [5, Proposition(19. 19), P. 512], M is cogenerator andhence by Proposition 5, M is a coretractable module.

Note that "for any module M and a cogenerator C, C⊕M is a cogenerator and so is a coretractablemodule, but M need not be coretractable module. So that Coretractability is not preserved by takingsubmodules, factor modules and direct summands". "However there are some special cases, butfirst recall that: a submodule S of an R-module M is called fully invariant if f(S) is contained in S forevery R-endomorphism f of M".

Proposition 8:"Let M=F⊕L be a coretractable R-module. If F is a fully invariant submoduleof M or F cogenerates M, then F is also coretrectable. In particular, if ⊕I F or ΠI F is coretractable forsome index set I, then so is F".

Consider M=Z⊕Q, EndZ(M)≅ (EndZHom(Q, Z)Hom(Z, Q) EndQ)=(Z 0Z Q), so for any φ ∈EndZ(M).

φ=(n 0m x), for some n, m∈Z and x∈Q. Then φ(Q)={ (n 0m x) (0y), y ∈Q}= { (0xy), x, y∈Q} ≅ Q.

Soφ(Q)⊆Q. Thus Q is fully invariant in M, but Q isn't coretrectable. Thus M is not coretractable by Proposition 8.

Proposition 9:" Let M 1, M 2, …, M n be coretractable modules, then so is ⊕i=1 n Mi".

As application of Proposition 9, each of the Z-module Z4⊕Z6⊕Z8, Z2⊕Z8, Zp∞⊕Zn (p is anyprime number, n∈Z+) is coretrectable.

Proposition 10: If M is module over commutative ring R such that [N:RM]= annM≠annN∀N ofM. Then every factor module of M is a coretractable module, where [N:RM]={r∈R: Mr⊆N} andannN={r∈R: Nr=0 }.

Proof: Suppose L/N<M/N, so N<L<M. By 3rd fundamental isomorphism theorem (M/N)/(L/N)≅M/L.Hence there exists an isomorphism g:(M/N)/(L/N)→M/L. Since annL≠annM, then there exists t∈annLand t∉annM. Define f:M/L→M by f(m+L)=mt. Clear f is well-defined and R-homomorphism.

Consider the sequence (M/N)/(L/N)g→M/Lf→Mπ→M/N; that is π°f°g:(M/N)/(L/N)→ M/N andπ°f°g[(M/N)/(L/N)] = π°f(M/L) = π(Mt) = (Mt+N)/N≠ 0M/N (since if Mt⊆N, then t∈[N:M]=annMwhich is a contradiction).

Proposition 11: If M is a module such that ∀S<M, ∃D≤⊕M such that S⊆D⊂M. Then M iscoretrectable.

Proof: Let S<M. By hypothesis S ⊆ D ⊂M for some direct summand D of M, there is f:M/D⟶M,f≠0. Define h: M/S⟶ M/D by h(x+S)=x+D∀x∈ M, then h is well-defined. Since f ≠ 0, there exists x+D∈M/D, x+D≠D and f(x+D)≠0; x∉D and hence x∉S. Thus x+S≠0 and h(x+S)=x+D≠ 0M/D =D. Now,f∘h: M/S⟶M and f∘h ≠0 since f∘h(x+S)=f(h(x+S))= f(x+D)≠ 0. Therefore M is a coretractablemodule.

Recall that (Schur's Lemma) stated " If M is simple module, then S=End(M) is division ring".

Proposition 12): An R-module M is simple if M is coretractable and EndR(M) is a division ring.

Proof: (⇒) Since M is simple, clearly M is coretractable and so EndR(M) is division ring by Schur'sLemma.

(⇐) Let S< M and S≠0. As M is coretrectable, ∃g ∈EndR(M), g≠0 such that g(S)=0, hence g is notone-one and that contradiction with EndR(M) is a division ring. Thus M is a simple module.

References

[1] Al-Saadi S A and Ibrahiem T A 2014 Strongly Rickart Rings Mathematical Theory and Modeling 4 8 pp 95-105

[2]Amini B, Ershad M and Sharif H 2009 Coretractable Modules J. Aust. Math. Soc 86 3 pp 289-304.

[3]Clark J, Lomp C, Vanaja N and Wisbauer R 2006 Lifting Modules Frontiers In Mathematics (Basel-Boston-Berlin:Birkhäuser Basel)

[4]Goodearl K R 1973 Ring Theory Nonsingular Rings And Modules

[5]Lam T Y 1999 Lectures on Modules and Rings (Springer, New York)

[6]Hadi I M A and Al-aeashi S N 2017 Strongly Coretractable Modules Iraqi Journal of Science 58 2C pp 1069-1075

[7]Hadi I M and Al-aeashi S N 2016 Strongly Coretractable Modules and Some Related ConceptsJournal of advances in Mathematics 12 12 pp 6881-88

[8]Hadi I M and Al-aeashi S N 2017 Y-coretractable and Strongly Y-Coretractable ModulesAsian Journal of Applied Sciences 5 2 pp 427-33

[9] Hadi I M and Al-aeashi S N 2017 P-coretractable and Strongly P-Coretractable Modules Asian Journal of Applied Sciences 5 2 pp 477-82

[10]Hadi I M and Al-aeashi S N 2017 Studying Some Results about Completely Coretractable Ring (CC-ring) Global Journal of Mathematics 10 1 644-47

[11]Hadi I M and Al-aeashi S N 2017 Some Results AboutCoretractable Modules Journal of AL-Qadisiyah for computer science and mathematics 9 2 pp 40-48

[12]Yaseen S M 2003 Coquasi-Dedekind Modules Ph. D. Thesis University Of Baghdad Baghdad Iraq